

Flavour constraints on scenarios with two or three heavy squark generations

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Abstract

We re-assess constraints from flavour-changing neutral currents in the kaon system on supersymmetric scenarios with a light gluino, two heavy generations of squarks and a lighter third generation. We compute for the first time limits in scenarios with three heavy squark families, taking into account QCD corrections at the next-to-leading order. We provide ready to use magic numbers for the computation of the Wilson coefficients at 2 GeV for these scenarios.

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1 Introduction

The initial searches for supersymmetric particles at the LHC indicate that minimal supersymmetric scenarios where all supersymmetric particles have similar masses below a TeV are not realized in nature. This, in addition to stringent bounds from supersymmetric contributions to flavour-changing neutral currents (FCNC), makes scenarios where some supersymmetric particles are significantly heavier than others more appealing, although these scenarios have been studied already for over a decade. With this motivation in mind and following the work in [1], we re-assess limits from FCNC in scenarios with two or three heavy families of squarks ($m_{\tilde{q}} \leq 10$ TeV), while keeping the gluino mass below 2 TeV. Studies of the QCD corrections in such setups, in particular for two heavy families, were performed in [2], [3] and [4, 5]. The first work considered leading-order (LO) QCD corrections and used the vacuum insertion approximation (VIA) for the hadronic matrix elements. The second one calculated next-to-LO (NLO) corrections and took into account lattice results for the bag parameters appearing in the matrix elements. More recently, [4] discussed a specific pattern of family symmetry breaking with hierarchical squarks and pointed out the need for a careful treatment of certain box diagrams in some cases, which was studied in more detail in [5].

The most dangerous supersymmetric contributions to FCNC often occur in the kaon sector, in particular contributions to Δm_K and the CP violation parameter ϵ . Our motivations for revisiting the corresponding limits are (i) significant progress in the determination of the experimental values and theoretical expectations in the Standard Model (SM) since the publication of [3], and (ii) the need for an NLO calculation of the QCD corrections in the case of three heavy squark generations.

We proceed as follows: in section 2, we summarize, for the sake of clarity, the way NLO QCD corrections are addressed for the $\Delta S = 2$ processes involving light gluinos and heavy scalars. We provide formulas that can be used to compute easily the renormalization group (RG) evolution of the Wilson coefficients relevant for $\Delta S = 2$ processes, both for the case of two heavy squark generations and for the case of three heavy families. We compare our results with the literature, finding that in some cases a change of basis was missing in earlier work. In section 3, we re-assess limits from Δm_K on the mass $m_{\tilde{q}}$ of the heavy families of down-type squarks, for given values of the flavour-violating parameters in the mass insertion (MI) approximation. We comment on the difference between the scenarios with three and two families of heavy scalars. In this section, we also determine bounds on the real and imaginary parts of flavour-violating parameters for given values of $m_{\tilde{q}}$ and the mass of the gluino, $m_{\tilde{g}}$.

2 QCD corrections for heavy squarks and a light gluino

2.1 Renormalization group evolution of the Wilson coefficients

The effective Hamiltonian for $\Delta S = 2$ transitions can be written as

$$H^{\Delta S=2} = \sum_{i=1}^5 C_i O_i + \sum_{i=1}^3 \tilde{C}_i \tilde{O}_i, \quad (1)$$

where the operators O_i are

$$\begin{aligned} O_1 &= \bar{d}^\alpha \gamma_\mu P_L s^\alpha \bar{d}^\beta \gamma^\mu P_L s^\beta, & O_2 &= \bar{d}^\alpha P_L s^\alpha \bar{d}^\beta P_L s^\beta, \\ O_3 &= \bar{d}^\alpha P_L s^\beta \bar{d}^\beta P_L s^\alpha, & O_4 &= \bar{d}^\alpha P_L s^\alpha \bar{d}^\beta P_R s^\beta, \\ O_5 &= \bar{d}^\alpha P_L s^\beta \bar{d}^\beta P_R s^\alpha, \end{aligned} \quad (2)$$

and $P_{L,R}$ are chirality projection operators, α, β are color indices, $\tilde{O}_i = O_i (L \leftrightarrow R)$, $\langle \tilde{O}_i \rangle = \langle O_i \rangle$, and $\tilde{C}_i = C_i (L \leftrightarrow R)$. The RG evolution of the Wilson coefficients from a high-energy scale M to a lower energy μ is determined by the 5×5 evolution matrix $\widehat{W}[\mu, M]$,

$$\mathbf{C}(\mu) = \widehat{W}[\mu, M] \mathbf{C}(M), \quad (3)$$

where \mathbf{C} represents a column vector with the 5 components C_i . We denote row vectors with an arrow, such as the row vector \vec{O} built up by the components O_i . At LO and NLO, respectively, $\widehat{W}[\mu, M]$ can be expressed as [6]

$$\begin{aligned} \widehat{W}[\mu, M]_{\text{LO}} &= \widehat{U}[\mu, M] \\ &= \left[\frac{\alpha_s(M)}{\alpha_s(\mu)} \right]^{\hat{\gamma}^{(0)T}/2\beta_0}, \\ \widehat{W}[\mu, M]_{\text{NLO}} &= \widehat{U}[\mu, M] + \frac{\alpha_s(\mu)}{4\pi} \hat{J}(\tilde{n}_f) \widehat{U}[\mu, M] - \frac{\alpha_s(M)}{4\pi} \widehat{U}[\mu, M] \hat{J}(\tilde{n}_f), \\ \beta_0 &= \frac{1}{3} (11N_c - 2\tilde{n}_f), \end{aligned} \quad (4)$$

where $N_c = 3$ is the number of colors and \tilde{n}_f equals the number of active fermion flavors, n_f , at energies below the gluino mass, $m_{\tilde{g}}$. At higher energies,

$$\tilde{n}_f = n_f + N_c + \frac{n_{\tilde{q}}}{4}, \quad (5)$$

where $n_{\tilde{q}}$ is the number of light squarks with a mass similar to $m_{\tilde{g}}$ [3]. The 5×5 matrices \hat{J} were calculated in [6]. They depend on the renormalization scheme. We use

the LRI scheme because it is also used in the lattice determination of the low-energy matrix elements. The one-loop anomalous dimension matrix (ADM), $\gamma^{(0)}$, is scheme-independent. It depends only on N_c and is a 5×5 matrix, since the five operators O_i entering the Hamiltonian (1) do not mix with others [7] during the evolution down to low energies.

In [6], the ADMs and \hat{J} are given in the Fierz basis of operators O_i^+ , defined by

$$\begin{aligned} O_1 &= O_1^+, \\ O_2 &= O_4^+, \\ O_3 &= -\frac{1}{2} \left(O_4^+ - \frac{1}{4} O_5^+ \right), \\ O_4 &= O_3^+, \\ O_5 &= -\frac{1}{2} O_2^+. \end{aligned} \tag{6}$$

Then we can define a matrix V to transform between the basis of the operators O_i and the one of the operators O_i^+ ,

$$\begin{aligned} \vec{O}^+ &= \vec{O} V^{-1}, \\ \mathbf{C}^+ &= V \mathbf{C}, \\ V &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 \end{pmatrix}. \end{aligned} \tag{7}$$

We will denote all evolution matrices in the Fierz basis with a tilde instead of a hat,

$$\begin{aligned} \tilde{U}[\mu, M] &= V^{-1} \hat{U}[\mu, M] V, \\ \tilde{W}[\mu, M] &= V^{-1} \hat{W}[\mu, M] V. \end{aligned} \tag{8}$$

In the scenario under consideration, the RG evolution of the Wilson coefficients starts at the mass scale of the heavy squarks, $M = m_{\tilde{q}}$, and ends at $\mu = 2 \text{ GeV}$, where the matrix elements of the $\Delta S = 2$ operators are calculated. Along the way, the gluinos and heavy quarks have to be integrated out at their mass scales. In the case of two heavy squark generations, we assume the lighter squarks to have a similar mass as the gluino. Hence, they are integrated out at $m_{\tilde{g}}$, too. The corresponding mass scales are depicted in Fig. 1. The figure also shows the matrices governing the evolution between the scales. The NLO evolution from $m_{\tilde{q}}$ down to μ yields

$$\begin{aligned} \mathbf{C}(\mu) &= V^{-1} \tilde{W}[\mu, m_{\tilde{q}}] V \mathbf{C}(m_{\tilde{q}}), \\ \tilde{W}[\mu, m_{\tilde{q}}] &= \tilde{W}[\mu, m_b] \tilde{W}[m_b, m_t] \tilde{W}[m_t, m_{\tilde{g}}] \tilde{W}[m_{\tilde{g}}, m_{\tilde{q}}] \Big|_{\text{NLO}}. \end{aligned} \tag{9}$$

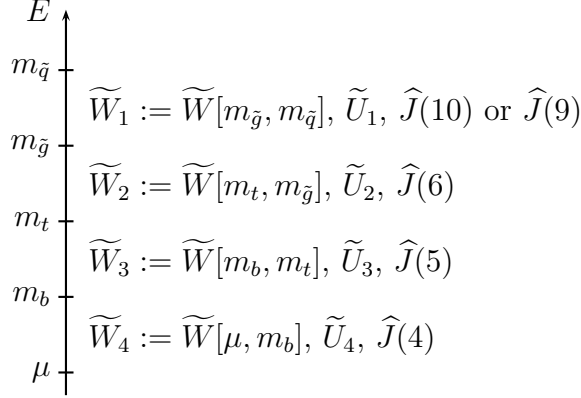


Figure 1: Energy thresholds, evolution matrices and $\hat{J}(\tilde{n}_f)$ involved in the RG evolution of the Wilson coefficients. In the energy range between $m_{\tilde{g}}$ and $m_{\tilde{q}}$, $\hat{J}(10)$ and $\hat{J}(9)$ refer to the case of 2 and 3 heavy squark generations, respectively. In \tilde{W}_4 we take $\mu = 2 \text{ GeV}$.

The LO evolution is analogous, with the replacement $\tilde{W} \rightarrow \tilde{U}$. The subscript “NLO” reflects the fact that we have to truncate the expansion of the product of evolution matrices such that it contains only terms up to $\mathcal{O}(\alpha_s)$ [8]. Explicitly,

$$\begin{aligned} \tilde{W}[\mu, m_{\tilde{q}}] &= \tilde{U}_4 \tilde{U}_3 \tilde{U}_2 \tilde{U}_1 + \tilde{U}_4 \tilde{U}_3 \tilde{U}_2 (\tilde{W}_1 - \tilde{U}_1) + \tilde{U}_4 \tilde{U}_3 (\tilde{W}_2 - \tilde{U}_2) \tilde{U}_1 \\ &\quad + \tilde{U}_4 (\tilde{W}_3 - \tilde{U}_3) \tilde{U}_2 \tilde{U}_1 + (\tilde{W}_4 - \tilde{U}_4) \tilde{U}_3 \tilde{U}_2 \tilde{U}_1. \end{aligned} \quad (10)$$

$\tilde{W}[\mu, m_{\tilde{q}}]$ depends on the values of α_s at the scales $m_{\tilde{q}}$, $m_{\tilde{g}}$, m_t , m_b and μ . Plugging in the known values of $\alpha_s(m_t)$, $\alpha_s(m_b)$ and $\alpha_s(\mu)$, we can write the evolution matrices $\tilde{W}[\mu, m_{\tilde{g}}]$ and $\tilde{W}[m_{\tilde{g}}, m_{\tilde{q}}]$ as functions of the free parameters $\alpha_s(m_{\tilde{g}})$ and $\alpha_s(m_{\tilde{q}})$ as well as the so-called magic numbers, analogously to Eqs. (9) of [3],

$$\begin{aligned} \tilde{W}[\mu, m_{\tilde{g}}] &= \sum_{r=1}^5 \left[b_{\text{LO}}^{(r)} + b_{\text{NLO}}^{(r)} + \eta_6 c^{(r)} \right] \eta_6^{a_r}, \\ \tilde{W}[m_{\tilde{g}}, m_{\tilde{q}}] &= \sum_{r=1}^5 \left[d^{(r)} + \eta_6 e^{(r)} + \eta_{\tilde{g}} \eta_6 f^{(r)} \right] \eta_{\tilde{g}}^{a'_r}, \end{aligned} \quad (11)$$

where

$$\eta_6 = \frac{\alpha_s(m_{\tilde{g}})}{\alpha_s(m_t)}, \quad \eta_{\tilde{g}} = \frac{\alpha_s(m_{\tilde{q}})}{\alpha_s(m_{\tilde{g}})}, \quad (12)$$

and the magic numbers $b^{(r)}$, $c^{(r)}$, $d^{(r)}$, $e^{(r)}$ and $f^{(r)}$ are 5×5 matrices. However, as the product $\tilde{W}[\mu, m_{\tilde{g}}] \tilde{W}[m_{\tilde{g}}, m_{\tilde{q}}]$ contains terms of order α_s^2 , to be consistent, we have to use the single evolution matrix

$$\tilde{W}[\mu, m_{\tilde{q}}] = \sum_{r,s} \left\{ \left[b_{\text{LO}}^{(r)} + b_{\text{NLO}}^{(r)} + \eta_6 c^{(r)} \right] d^{(s)} + b_{\text{LO}}^{(r)} \left[e^{(s)} + \eta_{\tilde{g}} f^{(s)} \right] \eta_6 \right\} \eta_6^{a_r} \eta_{\tilde{g}}^{a'_s} \quad (13)$$

rather than Eqs. (11). This approach is useful because for a given model, with three or two heavy families of squarks, we have to calculate only η_6 and $\eta_{\tilde{g}}$. Together with the magic numbers, Eq. (13) then immediately yields the values of the Wilson coefficients at $\mu = 2 \text{ GeV}$.

2.2 Evolution between squark and gluino mass scales

Due to the hierarchy between gluino and squark masses, integrating out all superparticles at the same scale would produce large logarithms. Therefore, we proceed in two steps, first integrating out the heavy squark generations at $m_{\tilde{q}}$ [2, 3] and then the gluinos and light squarks at $m_{\tilde{g}}$.

The matching at $m_{\tilde{q}}$ is visualized in Fig. 2. In the full theory, FCNC in the neutral kaon system stem from the $\Delta S = 2$ box diagram (I), where the scalar lines represent squark mass eigenstates. Using the MI approximation, we work with flavour eigenstates and flavour-changing mass insertions, represented by dashed lines and crosses in diagrams (IIa,b), and consider only diagram (IIa). This diagram contains only \tilde{d} and \tilde{s} squarks. Squarks of the third generation appear in box diagrams with at least three MI, i.e., in diagram (IIb) and higher orders. Thus, they do not contribute to the matching in the considered approximation. Consequently, even in the case of light third-generation squarks, there are no contributions from diagrams involving one heavy and one light squark, which require a special treatment [4, 5].¹

In the effective theory below $m_{\tilde{q}}$ but above $m_{\tilde{g}}$, diagrams (IIIa) and (IIIb) of Fig. 2 yield FCNC. The former contains the $\Delta S = 2$ operators O_i and \tilde{O}_i , which are represented by the dot, and it is suppressed by $1/m_{\tilde{q}}^2$. The latter diagram is suppressed by $m_g^2/m_{\tilde{q}}^4$ and thus negligible [2].

Altogether, the final matching condition is (IIa)=(IIIa). The resulting expressions for the gluino contributions to the Wilson coefficients at high energy, $C_i^{\tilde{g}}(m_{\tilde{q}})$, are given in Eq. (6) of [1] in terms of the MI parameters δ^d and in the notation we use.² They were calculated in [9, 2] and are the same in both cases for the squark masses we consider.

From $m_{\tilde{q}}$ down to $m_{\tilde{g}}$, gluinos and (if present) light squarks influence the RG evolution of the Wilson coefficients by changing the running of α_s and the two-loop ADM of the $\Delta S = 2$ operators. The impact on the ADM stems only from loops with gluinos and light squarks in the gluon propagator. Consequently, it can be taken into account by using $\hat{J}(n_f)$ of [6], where only loops with fermions and gluons were considered, and replacing n_f by \tilde{n}_f [3], as mentioned above. Explicitly, $\tilde{n}_f = 9$ if all squarks are heavy. If only two squark generations are heavy, $n_{\tilde{q}} = 4$ squarks are light and thus $\tilde{n}_f = 10$.

¹This can be different in scenarios with additional assumptions that cause CP violation to originate only from diagrams involving third-generation squarks, for example, Minimal Flavour Violation [4].

²We define $(\delta_{XY}^d)_{ij} := \frac{(m_{dXY}^2)_{ij}}{\sqrt{(m_{dXY}^2)_{ii}(m_{dXY}^2)_{jj}}}$, where $X, Y \in \{L, R\}$ and where one has to use the soft mass squared matrices in the super-CKM basis, where Yukawa couplings are diagonal.

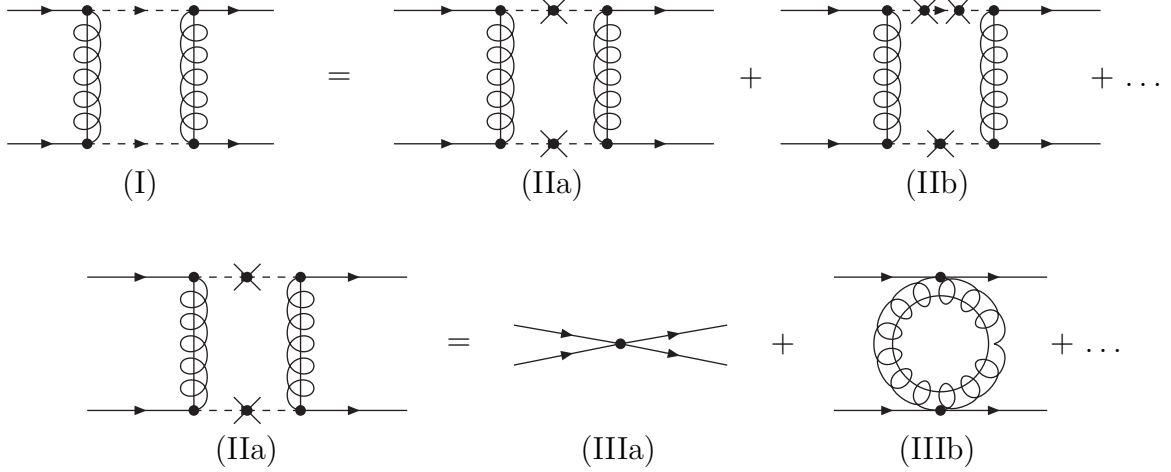


Figure 2: Diagrams representing the process of integrating out the heavy particles. The series of diagrams in the upper line represents the transition from the complete $\Delta S = 2$ box diagram to the mass insertion approximation, where the leading order is represented by diagram (IIa) and the next order by diagram (IIb). Crosses denote mass insertions. Diagrams (IIIa) and (IIIb) arise from diagram (IIa) when integrating out the d -type squarks.

Using these considerations and the appropriate formulas from the previous subsection, we calculated the magic numbers determining the NLO QCD corrections in both cases. We list them in Appendix B. Our results differ from those of [3]. As the disagreement in the magic numbers d , e and f vanishes for two heavy squark generations if we set $V = 1$, we conclude that [3] did not take into account the change of basis (7).

After the gluino and possible light squarks decouple at $m_{\tilde{g}}$, the system behaves as in [6, 7, 8]. Therefore, the magic numbers b and c should equal those given in [7], up to a different ordering of the components r and small differences due to changes in the experimental input parameters. We find larger differences than expected for some of the magic numbers. The main reason seems to be that $\mathcal{O}(\alpha_s^2)$ terms were not discarded in [7].

3 Limits on flavour- and CP-violating parameters from Δm_K and ϵ

3.1 Limits on the squark masses from Δm_K

In order to set a limit on the supersymmetric contribution to Δm_K , we write

$$\Delta m_K^T = \Delta m_K^{\text{SM}} + \Delta m_K^{\tilde{g}}, \quad (14)$$

where Δm_K^{T} represents the total theoretical value of Δm_K , while Δm_K^{SM} and $\Delta m_K^{\tilde{g}}$, respectively, represent the SM and the supersymmetric contributions due to \tilde{g} - \tilde{q} diagrams. Even in the SM, precise computations of Δm_K^{SM} are not possible due to unknown long-distance contributions [10]. Hence, the best one can do is to compare the best estimate obtained from the short-distance contributions [11], denoted by Δm_K^{SD} , to the experimental value

$$\begin{aligned}\Delta m_K^{\text{exp}} &= (3.483 \pm 0.0059) \times 10^{-15} \text{ GeV}, \\ \Delta m_K^{\text{SD}} &= (3.1 \pm 1.2) \times 10^{-15} \text{ GeV},\end{aligned}\tag{15}$$

where all quoted errors correspond to the 1σ C.L. Δm_K^{SD} was obtained by taking into account NNLO contributions from the charm quark. We can see that its central value already accounts for an 86 % of the experimental central value, but its uncertainty can easily account for the reported experimental value within the 1σ C.L. Given the lack of information on long-distance contributions, the best we can do to extract limits in extensions of the SM is to use $\Delta m_K^{\text{exp}} - \Delta m_K^{\text{SD}}$ as a constraint on the *order of magnitude* of $\Delta m_K^{\tilde{g}}$. In short, it cannot exceed 10^{-15} GeV, but lower limits based on whether or not $\Delta m_K^{\text{SD}} + \Delta m_K^{\tilde{g}}$ can actually be as large as the experimental value cannot be obtained. Furthermore, in some cases the importance of NLO corrections cannot be really appreciated because the differences between LO and NLO lie in the range of some units of 10^{-15} GeV. Therefore, we have to be cautious when using this for setting bounds in models for physics beyond the SM. As in the case of ϵ' , there are many models that can pretty easily yield a change of Δm_K by some units of 10^{-15} GeV, so meaningful constraints can indeed be obtained. However, we have to include a generous consideration of all the possible uncertainties in the calculation of $\Delta m_K^{\text{SD}} = 2 \text{Re} \langle K^0 | H_{\text{SM}}^{\Delta S=2} | \overline{K}^0 \rangle$. The form of this theoretical expression at the known QCD level can be found in [11]. For the gluino contribution, we have

$$\Delta m_K^{\tilde{g}} = 2 \text{Re} \langle K^0 | H_{\tilde{g}}^{\Delta S=2} | \overline{K}^0 \rangle = 2 \text{Re} \langle K^0 | \sum_{i=1}^5 C_i^{\tilde{g}} O_i + \sum_{i=1}^3 \tilde{C}_i^{\tilde{g}} \tilde{O}_i | \overline{K}^0 \rangle.\tag{16}$$

The Wilson coefficients at $\mu = 2 \text{ GeV}$ are determined by Eq. (13). The matrix elements of the operators are [7]

$$\begin{aligned}\langle K^0 | O_1 | \overline{K}^0 \rangle &= \frac{1}{3} M_K f_K^2 B_1(\mu), \\ \langle K^0 | O_i | \overline{K}^0 \rangle &= k_i \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2 B_i(\mu), \quad i = 2, 3, 4, 5, \\ k_i &= \frac{1}{8} \left(-\frac{5}{3}, \frac{1}{3}, 2, \frac{2}{3} \right)_i.\end{aligned}\tag{17}$$

These expressions differ by a factor of $\frac{1}{8M_K}$ from the corresponding ones in lattice studies such as [12, 13, 14, 15]. A factor $\frac{1}{4}$ stems from a different definition of the operators

and a factor $\frac{1}{2M_K}$ from the normalization of the kaon states. The definition (17) ensures that the numerical values of the bag parameters B_i are the same in both conventions. The VIA corresponds to

$$\begin{aligned}
\langle K^0 | O_1 | \bar{K}^0 \rangle_{\text{VIA}} &= \frac{1}{3} M_K f_K^2, \\
\langle K^0 | O_i | \bar{K}^0 \rangle_{\text{VIA}} &= k_i \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2, \quad i = 2, 3, \quad k_i = \frac{1}{8} \left(-\frac{5}{3}, \frac{1}{3} \right)_i, \\
\langle K^0 | O_4 | \bar{K}^0 \rangle_{\text{VIA}} &= \left[\frac{1}{24} + \frac{1}{4} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 \right] M_K f_K^2, \\
\langle K^0 | O_5 | \bar{K}^0 \rangle_{\text{VIA}} &= \left[\frac{1}{8} + \frac{1}{12} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 \right] M_K f_K^2.
\end{aligned} \tag{18}$$

In Appendix A we give the values of the experimental and the lattice parameters we use.

With the above points in mind, once we have the value of Δm_K^{SD} , its uncertainty, that we call $\sigma_{\Delta m_K^{\text{SD}}}$, the experimental value and its uncertainty, $\sigma_{\Delta m_K^{\text{exp}}}$, we can set a limit on the supersymmetric contribution, $\Delta m_K^{\tilde{g}}$, using

$$|\Delta m_K^{\tilde{g}}| < \left| \overline{\Delta m_K^{\text{exp}}} - \overline{\Delta m_K^{\text{SM}}} + 2(\sigma_{\Delta m_K^{\text{SD}}} + \sigma_{\Delta m_K^{\text{exp}}}) \right| = 2.8 \times 10^{-15} \text{ GeV}, \tag{19}$$

where $\overline{\Delta m_K^{\text{SD}}}$ and $\overline{\Delta m_K^{\text{exp}}}$ denote the central values.

In order to compare to [3], we consider the cases

$$\begin{aligned}
\text{I : } & (\delta_{\text{LL}}^d)_{12} = K, (\delta_{\text{LR}}^d)_{12} = (\delta_{\text{RL}}^d)_{12} = (\delta_{\text{RR}}^d)_{12} = 0, \\
\text{II : } & (\delta_{\text{LR}}^d)_{12} = K, (\delta_{\text{LL}}^d)_{12} = (\delta_{\text{RL}}^d)_{12} = (\delta_{\text{RR}}^d)_{12} = 0, \\
\text{III : } & (\delta_{\text{LL}}^d)_{12} = (\delta_{\text{RR}}^d)_{12} = K, (\delta_{\text{LR}}^d)_{12} = (\delta_{\text{RL}}^d)_{12} = 0, \\
\text{IV : } & (\delta_{\text{LR}}^d)_{12} = (\delta_{\text{RL}}^d)_{12} = K, (\delta_{\text{LL}}^d)_{12} = (\delta_{\text{RR}}^d)_{12} = 0.
\end{aligned} \tag{20}$$

In Fig. 3, we present the lower limits on the heavy squark masses $m_{\tilde{q}}$ according to Eq. (19) as a function of the mass of the gluino $m_{\tilde{g}}$, for a fixed value of the flavour parameter $K = 0.22$. The plots in Fig. 3 serve mainly (i) for comparison to Figs. 1 and 2 of [3]³, (ii) to re-assess the importance of the NLO QCD corrections and (iii) to understand the difference between the scenarios with two and three heavy families of squarks. We can see that the latter difference is not significant. In principle, this justifies using the QCD corrections of [3] also for scenarios with three heavy squark families, as done in [1] as a first approximation. However, as mentioned above, a change of basis was missing in [3]. Therefore, the limits shown in Fig. 3 are different from those in [3]. Nevertheless, the

³This is why we also plotted values of $m_{\tilde{g}}$ lower than the LHC bound around 900 GeV.

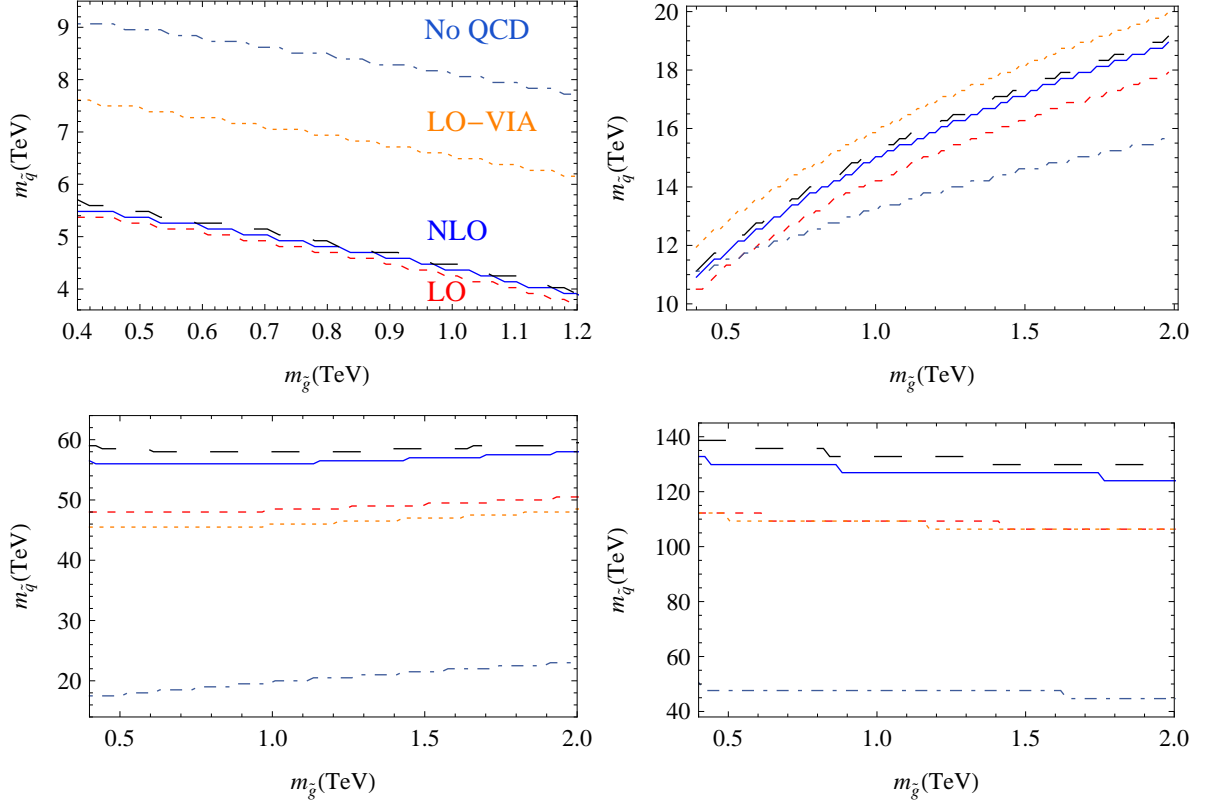


Figure 3: Lower limits on the squark masses for the different cases of Eq. (20) with $K = 0.22$, derived from Eq. (19). The blue continuous line corresponds to the NLO evolution, the medium-dashed (red) line corresponds to the LO approximation, the dotted (orange) line corresponds to the LO evolution using the VIA for the bag parameters and the dot-dashed (light blue) line to the results without QCD corrections. We have indicated the labels just for case I, but for the rest of the cases the coding is the same. For comparison, we have additionally plotted a long-dashed black line which represents the NLO evolution for the case where only two squark families are heavy.

qualitative behavior and order of magnitude of the limits are the same. Regarding the QCD corrections, each step of improving the accuracy (considering LO QCD corrections with VIA, including lattice bag parameters and finally taking into account NLO QCD corrections) can have a drastic effect. Only case II may be considered an exception, since here some of the corrections happen to cancel partially. We can also see that NLO QCD corrections, as opposed to just LO corrections, are indeed relevant, especially for cases III and IV.

We note that using different values of K can lower or raise considerably the limit on $m_{\tilde{q}}$. For example, for case III and $K = 0.1$, the limit becomes roughly $m_{\tilde{q}} > 27$ TeV [16], as opposed to $m_{\tilde{q}} > 56$ TeV for $K = 0.22$.

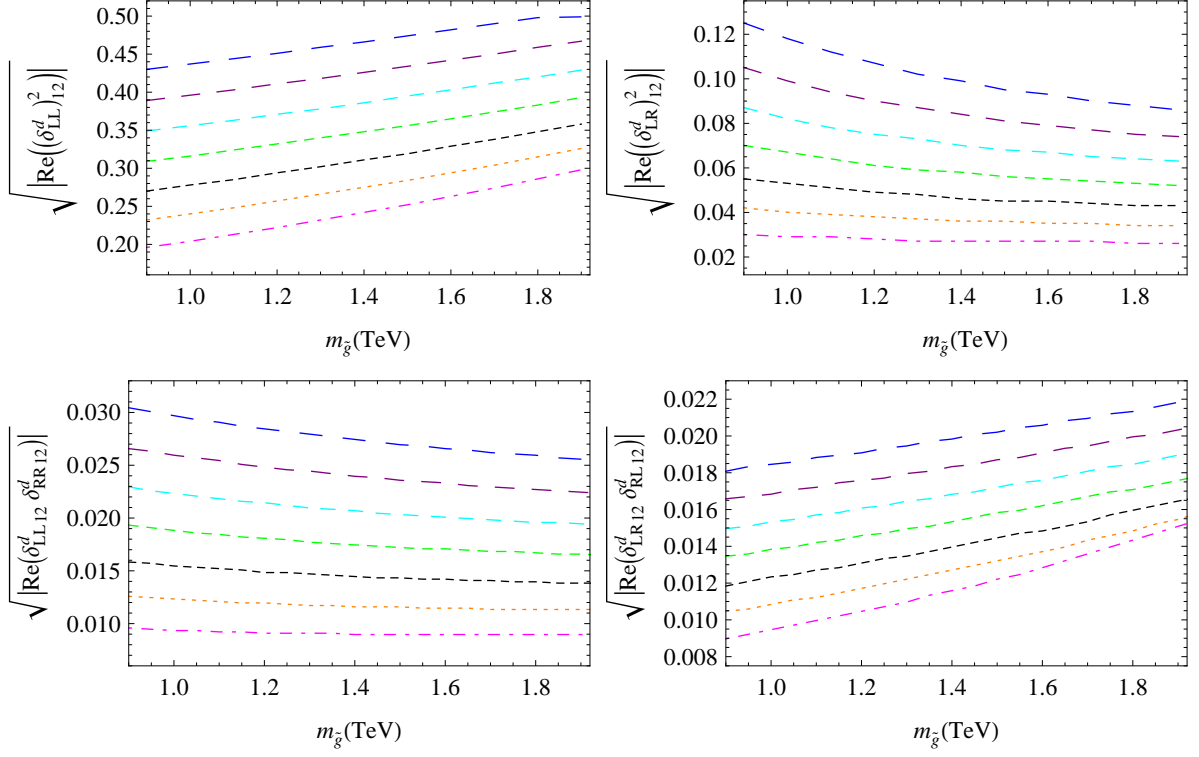


Figure 4: Upper limits on $\sqrt{|\text{Re}((\delta_{XY}^d)_{12}(\delta_{X'Y'}^d)_{12})|}$, derived using Δm_K . In all cases only the flavour-violating parameters shown on the vertical axis have been set to non-zero values. The limits on $\sqrt{|\text{Re}(\delta_{RR}^d)_{12}^2|}$ and $\sqrt{|\text{Re}(\delta_{RL}^d)_{12}^2|}$ can be obtained by interchanging $R \leftrightarrow L$ in the upper two panels. The two lower panels show, respectively, the cases, where $\sqrt{|\text{Re}((\delta_{LL}^d)_{12}(\delta_{RR}^d)_{12})|}$ or $\sqrt{|\text{Re}((\delta_{LR}^d)_{12}(\delta_{RL}^d)_{12})|}$ are not zero. The different curves correspond, from bottom to top, to $m_{\bar{q}} = 4, \dots, 10$ TeV.

3.2 Limits on mass insertion parameters

From Δm_K In Fig. 4 we present curves for fixed values of $m_{\bar{q}}$, plotting $m_{\tilde{g}}$ against the upper limits on $\sqrt{|\text{Re}((\delta_{XY}^d)_{12}(\delta_{X'Y'}^d)_{12})|}$ determined by requiring a sufficiently small gluino contribution to Δm_K , Eq. (19). The different curves correspond, from bottom to top, to $m_{\bar{q}} = 4, \dots, 10$ TeV. For $m_{\bar{q}} = 2$ TeV and the maximum value of the gluino mass we use, $m_{\tilde{g}} = 2$ TeV, the condition $m_{\bar{q}} \gg m_{\tilde{g}}$ defining the scenario under consideration would be violated.

The use of Δm_K indeed yields relevant limits on all MI parameters $(\delta_{XY}^d)_{12}$. They turn out to be strongest if $\text{Re}(\delta_{LR}^d)_{12}$ or $\text{Re}(\delta_{RL}^d)_{12}$ are non-zero. For cases when only the real part of one parameter is allowed to be non-zero and for gluino masses above 900 GeV, the limit is typically of order 10^{-1} and never below 0.02. Of course, the lower $m_{\bar{q}}$, the lower the limit. For the cases where either $\sqrt{|\text{Re}((\delta_{LL}^d)_{12}(\delta_{RR}^d)_{12})|}$ or $\sqrt{|\text{Re}((\delta_{LR}^d)_{12}(\delta_{RL}^d)_{12})|}$ are non-zero, the limits on the combinations are of $\mathcal{O}(10^{-2})$.

From ϵ As it is also well-known, we can extract limits on the imaginary parts of some combinations of the parameters $(\delta_{XY}^d)_{12}$ using the CP-violating parameter ϵ . Currently, its experimental and SM [11] values are, respectively,

$$\begin{aligned} |\epsilon| &= (2.228 \pm 0.011) \times 10^{-3}, \\ |\epsilon^{\text{NNLO}}| &= (1.81 \pm 0.28) \times 10^{-3}. \end{aligned} \quad (21)$$

Of course, in our case

$$|\epsilon| = \frac{|\text{Im} \langle K^0 | H_T^{\Delta S=2} | \bar{K}^0 \rangle|}{\sqrt{2} \Delta m_K} \quad (22)$$

where $H_T^{\Delta S=2} = H_{\text{SM}}^{\Delta S=2} + H_{\tilde{g}}^{\Delta S=2}$ with $H_{\tilde{g}}^{\Delta S=2}$ as given in Eq. (16). The limits are obtained using a condition analogous to Eq. (19), with the obvious replacements, yielding

$$|\epsilon^{\tilde{g}}| < 1.0 \times 10^{-3}. \quad (23)$$

Varying the mass of the gluino for fixed values of $m_{\tilde{q}}$, we found the upper limits on $\sqrt{|\text{Im}((\delta_{XY}^d)_{12}(\delta_{X'Y'}^d)_{12})|}$ that are presented in Fig. 5. As expected, the limits from ϵ prove to be indeed very strict. If only the parameter $\sqrt{|\text{Im}(\delta_{\text{LL}}^d)_{12}^2|}$ is not zero, the limits, varying according the mass of the squarks, are of order 10^{-2} . If only $\sqrt{|\text{Im}(\delta_{\text{LR}}^d)_{12}^2|}$ is not zero, the limits decrease to $\mathcal{O}(10^{-3})$. If two MI parameters are non-zero, the corresponding limit is of order 10^{-3} and always smaller than for only one non-vanishing parameter. Again, the lower the mass $m_{\tilde{q}}$, the lower the limits.

4 Conclusions

Supersymmetric scenarios involving light gluinos and heavy squarks have lately regained a considerable amount of attention, since they may be a way to realize supersymmetry in agreement with the LHC exclusion limits and with an acceptable amount of tuning. As it is also well-known, in such scenarios dangerous FCNC can occur especially in the kaon sector. In an earlier work [1], we considered a case with three heavy families of squarks with similar masses above 20 TeV. In order to estimate the gluino contributions to Δm_K and the CP-violating parameter ϵ , we used the evolution of Wilson coefficients from reference [3], where a scenario with two heavy families of squarks and a lighter third family with a mass comparable to that of the gluino was considered. In order to quantify precisely the accuracy of this estimate, we have reviewed the way NLO QCD corrections are addressed in supersymmetric scenarios involving heavy squarks and light gluinos. We have calculated the renormalization group evolution of the Wilson coefficients of the $\Delta S = 2$ operators at NLO for scenarios with two and three heavy squark families. This is codified in Eq. (13).

The SM determinations of Δm_K and ϵ have improved significantly over the last decade. Together with important improvements on the lattice QCD bag parameters

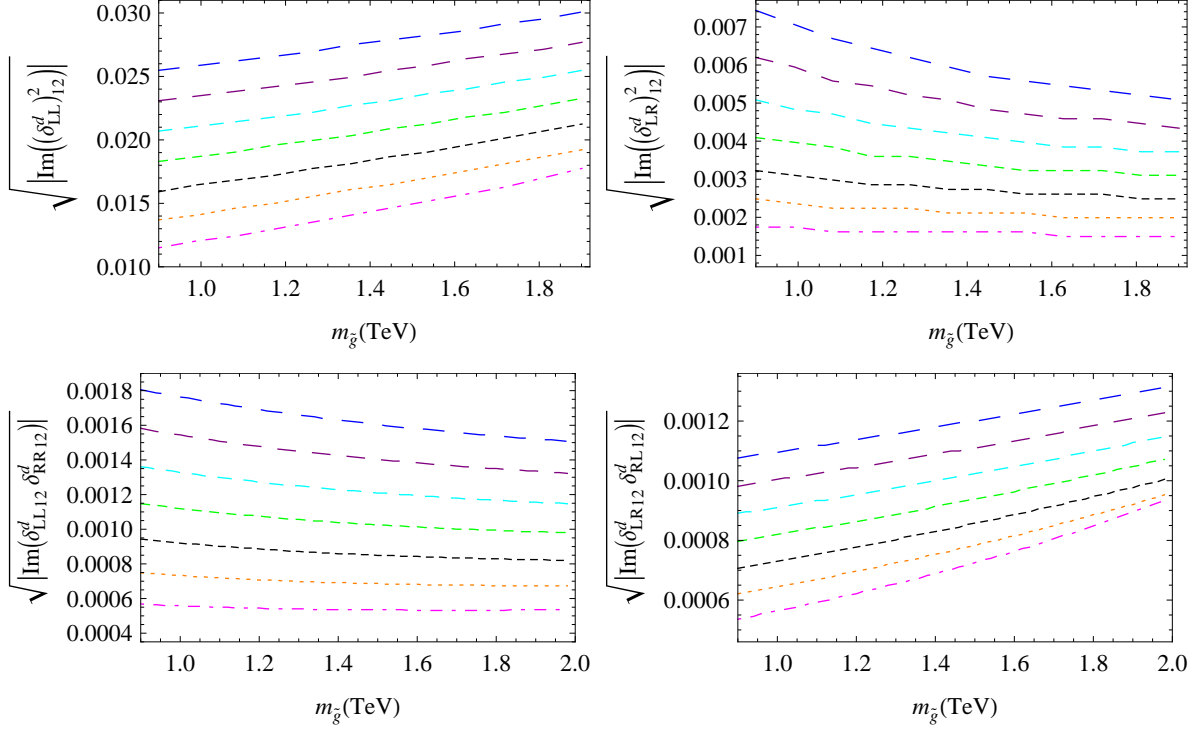


Figure 5: Upper limits on $\sqrt{|\text{Im}((\delta_{XY}^d)_{12}(\delta_{X'Y'}^d)_{12})|}$. The limits on $\sqrt{|\text{Im}(\delta_{RR}^d)_{12}|}$ and $\sqrt{|\text{Im}(\delta_{RL}^d)_{12}|}$ can be obtained by interchanging $R \leftrightarrow L$ in the corresponding upper two panels. The lower two panels correspond to the cases where only the combinations, respectively, $\sqrt{|\text{Im}((\delta_{LL}^d)_{12}(\delta_{RR}^d)_{12})|}$ or $\sqrt{|\text{Im}((\delta_{LR}^d)_{12}(\delta_{RL}^d)_{12})|}$ are non-zero. The different curves correspond, from bottom to top, to $m_{\tilde{q}} = 4, \dots, 10$ TeV.

entering the effective Hamiltonian of the $\Delta S = 2$ transitions, this proves relevant for re-assessing the limits on supersymmetric scenarios contributing to Δm_K and ϵ . This was the second motivation for this work.

We have then determined the lower limits on the mass of the heavy squarks, $m_{\tilde{q}}$, coming from Δm_K , using the MI approximation with a value of $\sqrt{|\text{Re}((\delta_{XY}^d)_{12}^2)|} = 0.22$, $X, Y \in \{L, R\}$, for one or two flavour-violating parameters. For cases with only one non-zero flavour-violating parameter, the limits on $m_{\tilde{q}}$ for a gluino mass of about 900 GeV and three heavy squark families are as follows: if only either $\sqrt{|\text{Re}((\delta_{LL}^d)_{12}^2)|} = 0.22$ or $\sqrt{|\text{Re}((\delta_{RR}^d)_{12}^2)|} = 0.22$ is non-zero, $m_{\tilde{q}}$ has to be larger than 5 TeV. If only either $\sqrt{|\text{Re}((\delta_{LR}^d)_{12}^2)|} = 0.22$ or $\sqrt{|\text{Re}((\delta_{RL}^d)_{12}^2)|} = 0.22$ is non-zero, the limit is above 14 TeV. If only the first two squark families are heavy, the limit changes by about 100 GeV. For the case of $\sqrt{|\text{Re}((\delta_{LL}^d)_{12}(\delta_{RR}^d)_{12})|} = 0.22$ and the case of $\sqrt{|\text{Re}((\delta_{LR}^d)_{12}(\delta_{RL}^d)_{12})|} = 0.22$, while the other flavour-violating parameters are zero, the limits are respectively around 56 TeV and 125 TeV, again for gluinos around 900 GeV. For two heavy families of

squarks, the limits are a few TeV larger than for three heavy families. We conclude that when the gluino contribution to Δm_K is close to its upper limit, it changes only by a few percent if the mass of the third-generation squarks varies between $m_{\tilde{g}}$ and $m_{\tilde{q}}$.

We have also noticed that indeed the NLO QCD evolution, as opposed to just the LO evolution, proves relevant, especially for cases where there is more than one type of flavour-violating parameter $(\delta_{XY}^d)_{12}$ involved [3]. As we have seen, the difference can be as large as 20 %.

Finally, we have obtained bounds on the real and imaginary parts of the combinations of flavour-violating parameters $(\delta_{XY}^d)_{12}$ that enter into the Wilson coefficients $C_i^{\tilde{g}}$ and $\tilde{C}_i^{\tilde{g}}$ for three heavy families of squarks. The former bounds stem from Δm_K and the latter from ϵ . We have explored these limits for squark masses between 4 and 10 TeV, varying the mass of the gluino for each value. In short, when only the real part of one type of parameter $(\delta_{XY}^d)_{12}$ is allowed to exist and for masses of the gluino above 900 GeV, the limit on $|\text{Re}(\delta_{XY}^d)_{12}|$ is at most of $\mathcal{O}(10^{-1})$ in most cases, of course the lower the mass $m_{\tilde{g}}$, the lower the limit. When only $\sqrt{|\text{Im}(\delta_{XY}^d)_{12}|^2}$ for one type of parameter is allowed to exist, its limit is of $\mathcal{O}(10^{-2})$ or smaller. Of course, when several types of flavour- and CP-violating parameters $(\delta_{XY}^d)_{12}$ are non-vanishing, the upper limits become smaller.

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A Experimental information

Values of experimental and lattice QCD parameters not given in the main part are listed in Tab. 1. We are aware that there have recently been some improvements in the determination of B_K [17, 18, 19, 20, 21, 22], which have been averaged to a value of 0.7643 ± 0.0097 [23]. This would change the values for Δm_K and ϵ obtained in [11], but we have checked that the impact on our results is negligible, since the limits are mainly determined by $\sigma_{\Delta m_K}$ and σ_ϵ .

Experimental & lattice inputs		
$\alpha_3(M_Z)$	0.1184 ± 0.0007	[24]
$m_t := M_t^{\text{Pole}}$	$(173.5 \pm 0.6 \pm 0.8) \text{ GeV}$	[24]
$m_b := m_b(m_b)$	$(4.18 \pm 0.03) \text{ GeV}$	[24]
f_K	$(0.1561 \pm 0.00085) \text{ GeV}$	[24]
M_K	$(0.497614 \pm 0.000022) \text{ GeV}$	[24]
B_1	0.51 ± 0.02	[15]
B_2	0.73 ± 0.04	[15]
B_3	1.29 ± 0.11	[15]
B_4	1.04 ± 0.07	[15]
B_5	0.76 ± 0.09	[15]
$m_s(2 \text{ GeV})^{\text{RI}}$	$(0.12 \pm 0.006) \text{ GeV}$	*
$m_d(2 \text{ GeV})^{\text{RI}}$	$(0.006 \pm 0.001) \text{ GeV}$	*

Table 1: Experimental and lattice QCD values used for our analysis. The bag parameters B_i are given in the RI scheme. *We have computed with *RunDec* [25] the values of m_s and m_d in the RI scheme, using as input the values given by the PDG [24] in the $\overline{\text{MS}}$ scheme.

B Magic numbers

B.1 Evolution below $m_{\tilde{g}}$

$$a_r = \left(\frac{2}{7}, -\frac{8}{7}, \frac{1}{7}, \frac{1 - \sqrt{241}}{21}, \frac{1 + \sqrt{241}}{21} \right)_r \approx (0.29, -1.1, 0.14, -0.69, 0.79)_r \quad (24)$$

$$\begin{aligned}
b_{\text{LO}11}^{(r)} &= (0.77, 0, 0, 0, 0)_r \\
b_{\text{LO}22}^{(r)} &= (0, 0, 0, 1.8, 0.0083)_r \\
b_{\text{LO}23}^{(r)} &= (0, 0, 0, -0.48, 0.13)_r \\
b_{\text{LO}32}^{(r)} &= (0, 0, 0, -0.12, 0.032)_r \\
b_{\text{LO}33}^{(r)} &= (0, 0, 0, 0.032, 0.48)_r \\
b_{\text{LO}44}^{(r)} &= (0, 2.8, 0, 0, 0)_r \\
b_{\text{LO}45}^{(r)} &= (0, 0.94, -0.29, 0, 0)_r \\
b_{\text{LO}55}^{(r)} &= (0, 0, 0.88, 0, 0)_r \\
b_{\text{NLO}11}^{(r)} &= (0.045, 0, 0, 0, 0)_r \\
b_{\text{NLO}22}^{(r)} &= (0, 0, 0, 0.51, 0.0020)_r \\
b_{\text{NLO}23}^{(r)} &= (0, 0, 0, -0.13, 0.030)_r \\
b_{\text{NLO}32}^{(r)} &= (0, 0, 0, 0.088, -0.0036)_r
\end{aligned} \quad (25)$$

$$\begin{aligned}
b_{\text{NLO}33}^{(r)} &= (0, 0, 0, -0.023, -0.055)_r \\
b_{\text{NLO}44}^{(r)} &= (0, 1.3, 0, 0, 0)_r \\
b_{\text{NLO}45}^{(r)} &= (0, 0.42, 0.096, 0, 0)_r \\
b_{\text{NLO}54}^{(r)} &= (0, 0.14, 0, 0, 0)_r \\
b_{\text{NLO}55}^{(r)} &= (0, 0.048, -0.063, 0, 0)_r
\end{aligned} \tag{26}$$

$$\begin{aligned}
c_{11}^{(r)} &= (-0.015, 0, 0, 0, 0)_r \\
c_{22}^{(r)} &= (0, 0, 0, -0.19, -0.0026)_r \\
c_{23}^{(r)} &= (0, 0, 0, -0.015, 0.0063)_r \\
c_{32}^{(r)} &= (0, 0, 0, 0.012, -0.0099)_r \\
c_{33}^{(r)} &= (0, 0, 0, 0.00096, 0.024)_r \\
c_{44}^{(r)} &= (0, -0.51, 0.0054, 0, 0)_r \\
c_{45}^{(r)} &= (0, -0.26, -0.0061, 0, 0)_r \\
c_{54}^{(r)} &= (0, 0, -0.016, 0, 0)_r \\
c_{55}^{(r)} &= (0, 0, 0.018, 0, 0)_r
\end{aligned} \tag{27}$$

B.2 Evolution above $m_{\tilde{g}}$

$$\begin{aligned}
d_{11}^{(r)} &= (1.0, 0, 0, 0, 0)_r \\
d_{22}^{(r)} &= (0, 0, 0, 0.98, 0.017)_r \\
d_{23}^{(r)} &= (0, 0, 0, -0.26, 0.26)_r \\
d_{32}^{(r)} &= (0, 0, 0, -0.064, 0.064)_r \\
d_{33}^{(r)} &= (0, 0, 0, 0.017, 0.98)_r \\
d_{44}^{(r)} &= (0, 1.0, 0, 0, 0)_r \\
d_{45}^{(r)} &= (0, 0.33, -0.33, 0, 0)_r \\
d_{55}^{(r)} &= (0, 0, 1.0, 0, 0)_r
\end{aligned} \tag{28}$$

These magic numbers correspond to LO effects. Consequently, they are independent of \tilde{n}_f , in particular of the number of heavy squarks.

B.2.1 Three heavy squark generations

$$a'_r = \left(\frac{2}{5}, -\frac{8}{5}, \frac{1}{5}, \frac{1 - \sqrt{241}}{15}, \frac{1 + \sqrt{241}}{15} \right)_r \approx (0.40, -1.6, 0.20, -0.97, 1.1)_r \tag{29}$$

$$\begin{aligned}
e_{11}^{(r)} &= (0.0019, 0, 0, 0, 0)_r \\
e_{22}^{(r)} &= (0, 0, 0, 0.15, 0.00044)_r \\
e_{23}^{(r)} &= (0, 0, 0, -0.040, 0.0067)_r \\
e_{32}^{(r)} &= (0, 0, 0, 0.013, -0.0059)_r \\
e_{33}^{(r)} &= (0, 0, 0, -0.0034, -0.091)_r \\
e_{44}^{(r)} &= (0, 0.27, 0, 0, 0)_r \\
e_{45}^{(r)} &= (0, 0.091, 0.037, 0, 0)_r \\
e_{54}^{(r)} &= (0, 0.018, 0, 0, 0)_r \\
e_{55}^{(r)} &= (0, 0.0061, -0.035, 0, 0)_r \\
f_{11}^{(r)} &= (-0.0019, 0, 0, 0, 0)_r \\
f_{22}^{(r)} &= (0, 0, 0, -0.15, -0.0044)_r \\
f_{23}^{(r)} &= (0, 0, 0, 0.0088, 0.025)_r \\
f_{32}^{(r)} &= (0, 0, 0, 0.0098, -0.017)_r \\
f_{33}^{(r)} &= (0, 0, 0, -0.00058, 0.095)_r \\
f_{44}^{(r)} &= (0, -0.28, 0.0061, 0, 0)_r \\
f_{45}^{(r)} &= (0, -0.12, -0.0098, 0, 0)_r \\
f_{54}^{(r)} &= (0, 0, -0.018, 0, 0)_r \\
f_{55}^{(r)} &= (0, 0, 0.029, 0, 0)_r
\end{aligned} \tag{30}$$

B.2.2 Two heavy squark generations

$$a'_r = \left(\frac{6}{13}, -\frac{24}{13}, \frac{3}{13}, \frac{1 - \sqrt{241}}{13}, \frac{1 + \sqrt{241}}{13} \right)_r \approx (0.46, -1.8, 0.23, -1.1, 1.3)_r \tag{32}$$

$$\begin{aligned}
e_{11}^{(r)} &= (-0.012, 0, 0, 0, 0)_r \\
e_{22}^{(r)} &= (0, 0, 0, 0.19, -0.00026)_r \\
e_{23}^{(r)} &= (0, 0, 0, -0.050, -0.0040)_r \\
e_{32}^{(r)} &= (0, 0, 0, 0.010, -0.0084)_r \\
e_{33}^{(r)} &= (0, 0, 0, -0.0027, -0.13)_r \\
e_{44}^{(r)} &= (0, 0.35, 0, 0, 0)_r \\
e_{45}^{(r)} &= (0, 0.12, 0.038, 0, 0)_r
\end{aligned}$$

$$\begin{aligned}
e_{54}^{(r)} &= (0, 0.018, 0, 0, 0)_r \\
e_{55}^{(r)} &= (0, 0.0060, -0.042, 0, 0)_r \\
f_{11}^{(r)} &= (0.012, 0, 0, 0, 0)_r \\
f_{22}^{(r)} &= (0, 0, 0, -0.19, -0.0038)_r \\
f_{23}^{(r)} &= (0, 0, 0, 0.020, 0.035)_r \\
f_{32}^{(r)} &= (0, 0, 0, 0.012, -0.014)_r \\
f_{33}^{(r)} &= (0, 0, 0, -0.0013, 0.13)_r \\
f_{44}^{(r)} &= (0, -0.35, 0.0060, 0, 0)_r \\
f_{45}^{(r)} &= (0, -0.14, -0.012, 0, 0)_r \\
f_{54}^{(r)} &= (0, 0, -0.018, 0, 0)_r \\
f_{55}^{(r)} &= (0, 0, 0.036, 0, 0)_r
\end{aligned} \tag{33}$$

$$\tag{34}$$

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